

# Inflation in warped geometries

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## Abstract

We argue that brane anti-brane inflation in string theory de-Sitter vacua of Kachru-Kallosh-Linde-Trivedi (KKLT) is captured by the dynamics of a  $D3$ -brane probe in the local KKLT model constructed in hep-th/0203041. This provides a framework to study in a controllable way corrections to the inflationary slow roll parameter  $\eta$  due to conformal symmetry breaking in a warped geometry throat. We compute the leading correction to  $\eta$  for the inflation in the Klebanov-Tseytlin throat geometry. We find that in certain regime this correction tends to decrease  $\eta$ . Computations in a different regime suggest however that it is unlikely that  $\eta \ll 1$  can be achieved with the  $D3$ -brane throat inflation.

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# 1 Introduction

An important question in string theory remains to find an explicit realization (in a single model) of two main features of our Universe's cosmic evolution: early inflation and the present-day acceleration. Progress in this direction has recently been reported in [1]. Using an effective four-dimensional description, the authors discussed embedding of  $D3-\overline{D3}$  inflation in the string theory de-Sitter vacua of [2] (KKLT). KKLT de-Sitter vacua can be understood by starting with string theory flux compactifications of [6] (GKP), which provides a global description (compactification) of a local (non-compact) Klebanov-Strassler (KS) model [7]. GKP compactifications lead to four dimensional *no-scale* models [3–5]: a vanishing 4d cosmological constant and a complex Kähler modulus  $\rho$  (related to the overall size of the compactification manifold). In [2] it was pointed out that string theory instanton corrections [8] modify the no-scale structure of GKP, leading to a supersymmetric  $AdS_4$  vacuum with fixed  $\rho = \rho_c$ . It was further argued that, for suitable choices of parameters, the  $\overline{D3}$  brane lifts the  $AdS_4$  vacuum to a  $dS_4$  background, while keeping stabilized all moduli of the compactification manifold. The  $\overline{D3}$  moduli are stabilized as well, as in the warped geometry of the compactification manifold it is driven to a point where the warp factor has local minimum<sup>1</sup>. Lastly, the inflation in the KKLT vacua is realized by placing a  $D3$ -brane (a *probe*) in the throat region of the compactification manifold [1]. In this inflationary model the inflaton field  $\phi$  of the effective four dimensional description is represented by the separation between the probe brane and the  $\overline{D3}$  brane, stabilized at the end of the throat. Unfortunately, it was argued that the slow roll parameter associated with the  $\phi$ -field inflation is too large for this model to be realistic

$$\eta \equiv \frac{1}{3} \frac{V_{inf}(\phi)''}{H^2} = \frac{2}{3}. \quad (1.1)$$

In (1.1)  $V_{inf}(\phi)$  is the inflaton potential obtained after integrating out the Kähler modulus of the compactification manifold

$$V_{inf} = V(\rho, \phi) \Big|_{\rho=\rho_c}, \quad (1.2)$$

and  $H$  is the Hubble scale of the de-Sitter vacua. It was suggested [1] that the generic  $\eta$ -problem (1.1) might be alleviated once the  $\phi$ -dependence of the  $\rho$ -modulus superpotential is taken into account, or if a Kähler stabilization mechanism (as opposite to the

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<sup>1</sup>This occurs at the end of the Klebanov-Strassler throat of the GKP compactification.

superpotential stabilization) is used to fix  $\rho$ . However, it is not known whether either of these mechanisms actually works: the  $\phi$ -dependence of the effective four dimensional superpotential is not known at present and, though some of the  $\alpha'$  corrections that break the no-scale structure of GKP are computed [9], the reported corrections do not lead to the suggested Kähler stabilization mechanism.

In this paper we explain a different framework for analyzing inflation in warped de-Sitter string theory geometries which, in particular, bypasses the difficulties of computing corrections to  $\eta$  from the effective four dimensional perspective mentioned above. We observe that for some aspects of the brane inflation deep inside the warped throat geometries the details of the compactification manifold, which provides a UV completion of the otherwise infinite throat, are irrelevant. All that matters from the compactification manifold is that it supplies a four dimensional Hubble parameter  $H$ . Also, in this setup it is assumed that all moduli of the compactification manifold are fixed, and the scale of moduli stabilization  $E_s$  is much higher than the relevant scales of inflation  $E_s \gg H$ ,  $E_s \gg |\phi|$ . It is clear that brane inflation in this class of models is equivalent to the  $D3$  probe brane dynamics in the local geometry where the throat, rather than terminating on some complicated (compact) Calabi-Yau manifold, extends to infinity. The advantage of this viewpoint is that, unlike compact KKLT or GKP backgrounds, the corresponding local models can be rather easily and explicitly constructed. For example, much like KS model [7] is a local description of the throat geometry of the GKP compactification, the de-Sitter deformed KT model [10] described in [11, 12] is a local realization of the throat geometry of the KKLT model<sup>2</sup>. The inflation, or equivalently the brane probe dynamics, can now be studied very explicitly and analytically.

In the next section we review the effective four dimensional approach to inflation in KKLT de-Sitter vacua [1]. In section 3, after explaining the general relation between effective four dimensional inflation and higher dimensional probe brane dynamics, we show that the  $\eta$ -problem (1.1) arises in the leading approximation from the  $D3$  probe dynamics in  $AdS_5 \times T^{1,1}$  (KW model [14]), with  $AdS_5$  written in hyperbolic coordinates (or de-Sitter slicing). We then study  $D3$  probes in the de-Sitter deformed KT geometry and demonstrate that in certain regime  $\eta < \frac{2}{3}$ . We point out that it is unlikely that

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<sup>2</sup>Strictly speaking, the correct local model would be de-Sitter deformation of the Klebanov-Strassler solution [7]. For the inflation occurring far from the end of the KS throat the difference between KT and KS models is subdominant, as it will be for their corresponding de-Sitter deformations. KS de-Sitter deformation as proposed in [12] can be explicitly constructed [13].

$\eta \ll 1$  can be achieved in this class of inflationary models.

We would like to emphasize that though in this paper we study inflation as a probe dynamics in de-Sitter deformed KT backgrounds [11,12], our approach is quite general. Following the prescription given in [12], de-Sitter deformation can be constructed in principle for *any* RG flow realizing supergravity dual to a four dimensional gauge theory. Some deformations of this type are discussed in [15,16]. It is possible to study analytically the probe dynamics in these backgrounds, and try to address the question whether always  $\eta < \frac{2}{3}$ , and in particular, whether it is possible to achieve  $\eta \ll 1$ . Also, one can study more exotic inflationary models, as a probe dynamics of a  $Dp$ -brane, for  $p > 3$ , wrapping a  $(p-3)$  cycle of the transverse manifold. An example of this would be a five-brane wrapping a two-cycle of the de-Sitter model described in section 3 of [15].

## 2 4d effective description of inflation

Consider a set of parallel branes placed at different positions in their common transverse space. Their relative position is described by some matrix-valued scalar field living on their world volume. Under suitable circumstances this theory also contains dynamical gravity. The idea of  $D - \overline{D}$  inflation [17] is that in such a system of branes and anti-branes, the field describing their separation can act as an inflaton in the world volume theory. Because the system is not BPS, the branes experience a net attractive force and thus the would-be inflaton has a nontrivial potential. For branes in flat space this potential leads to a large slow roll parameter, which makes this scenario somewhat unsuitable for sustained slow roll inflation.

This problem is slightly alleviated by embedding [1] the mechanism described above in a warped compactification, the intuition being that the warping of the geometry leads to an inflaton potential which is flatter than in the case of branes in flat space. This intuition is realized if one ignores the stabilization of the modulus corresponding to the size of the transverse space. Taking it into account modifies this conclusion and leads to fast roll inflation [1]. We will now briefly review these features.

The KKLT scenario is based on a compact geometry. To leading order, the local geometry (Klebanov-Strassler throat) can be approximated by  $AdS_5 \times X^5$ , with the usual assumptions leading to small curvature:

$$ds^2 = h(r)^{-1/2}(-dt^2 + d\vec{x}^2) + h(r)^{1/2}(dr^2 + r^2 ds_{X^5}^2), \quad h(r) = \frac{R^4}{r^4}. \quad (2.1)$$

At the IR and UV ends of the geometry there are significant departures from this structure. In particular, in the IR the geometry is smoothly cutoff before  $h$  blows up; this leads to the non-decoupling of gravity and the existence of a standard Einstein-Hilbert action in the four-dimensional effective action.

Due to the cancellation between the gravitational attraction and the electrostatic repulsion due to the RR flux, a  $D3$  brane experiences no force. A  $\overline{D3}$  brane however is dynamically localized at the IR end of the AdS throat. Combining these two effects leads to a relatively shallow potential felt by the brane, which is entirely due to the presence of the anti-brane and is substantially different from the flat space one due to the warping of the geometry. This potential, which in the compact model appears in the four-dimensional effective action, is computed in a probe approximation; denoting by  $r_0$  and  $r_1$  the location of the anti-brane and the distance to the brane, respectively, the potential turns out to be [1]

$$V = 2T_3 \frac{r_0^4}{R^4} \left( 1 - \frac{1}{N} \frac{r_0^4}{r_1^4} \right). \quad (2.2)$$

In explicit string theory realizations of this scenario the coefficient  $r_0/R$  is exponentially small and thus (2.2) leads to the required flat inflaton potential.

In the presence of a stabilized radial modulus the situation is somewhat different and [1] argues that the  $\eta$ -parameter is always of order unity. At the level of the effective action this is due to an unfortunate interplay between the would-be inflaton  $\phi$  (the analog of  $r_1$  in equation (2.2)) describing the position of the  $D3$  brane in the KS throat, the four-dimensional field  $\rho$  associated to the Kähler form and the correct identification of the actual volume modulus  $r$ . As argued in [18], the four-dimensional Kähler potential of the combined  $(\phi, \rho)$  system is

$$K(\rho, \phi) = -3 \ln(\rho + \bar{\rho} - k(\phi, \bar{\phi})), \quad (2.3)$$

which leads to the identification of the actual volume modulus  $r$  as

$$2r = \rho + \bar{\rho} - k(\phi, \bar{\phi}). \quad (2.4)$$

The complete potential now has two possible rolling directions. For artificially fixed volume modulus the surviving direction exhibits slow roll. If however, the volume modulus is dynamically fixed using a superpotential-based stabilization mechanism, the surviving direction exhibits fast roll<sup>3</sup>. This is mainly due to the fact that this

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<sup>3</sup>The problem reviewed here can in principle be bypassed [1] by using a Kähler-based stabilization mechanism, but such compactifications are difficult to control.

mechanism yields an expectation value for  $\rho$  rather than  $r$ . The problem emerges from the fact that, as  $\rho$  acquires a nonvanishing expectation value, the would-be canonically normalized inflaton

$$\varphi = \phi \sqrt{6/(\rho + \bar{\rho})} \quad (2.5)$$

acquires a mass of the order of the vacuum energy which in turn leads to an  $\eta$ -parameter of order unity. Since in the following section we will describe an alternative to the discussion in [1], let us describe this in more detail.

Given an arbitrary superpotential which depends only on  $\rho$ , the potential for the combined  $(\phi, \rho)$  system is

$$V = \frac{1}{6r} \left[ \partial_\rho W \bar{\partial}_\rho W \left( 1 + \frac{1}{2r} \frac{\partial_\phi k \partial_{\bar{\phi}} k}{\partial_{\phi \bar{\phi}} k} \right) - \frac{3}{2r} (\bar{W} \partial_\rho W + W \bar{\partial}_\rho W) \right] + V_{anti-brane}. \quad (2.6)$$

In this equation, the square bracket arises as a consequence of the existence of the superpotential  $W$  while the last term is due to the presence of the anti-brane at the end of the throat. It is worth pointing out that this last term is analogous to the first term in (2.2) while the second term in that equation is part of the square bracket in the equation above.

Assuming that  $k = \phi \bar{\phi}$ , the potential (2.6) has a de-Sitter minimum at some  $\rho = \rho_c$  and vanishing  $\phi$  at which the potential takes the value  $V_c = V(\rho_c, 0)$  and defining the canonically-normalized field (2.5), it follows that [1]

$$V \simeq V_c \left( 1 + \frac{1}{3} \varphi \bar{\varphi} \right). \quad (2.7)$$

This implies that the “inflaton” field  $\varphi$  behaves similarly to a conformally coupled scalar in a space with cosmological constant  $V_c$

$$m_\varphi^2 = \frac{1}{3} V_c = H^2, \quad (2.8)$$

where  $H$  is the Hubble constant. This implies that  $\eta$  is of order unity and thus incompatible with sustained slow roll inflation.

In this effective field theory framework it is difficult to analyze how the inflaton potential responds to deviations in the geometry from the exact  $AdS$  throat. In the next section we will describe an alternative derivation of (2.8) which will allow us to compute such corrections.

### 3 Inflation as a probe dynamics

Recall that after integrating out all moduli of the compactification manifold  $\mathcal{M}_6$ , the low energy effective action is given by [2]

$$S_{bulk} = \frac{1}{2k_4^2} \int_{\mathcal{M}_4} d^4x \sqrt{-g} \left( R - 2\Lambda \right), \quad (3.1)$$

where  $\Lambda \equiv 3H^2 > 0$  is the four dimensional cosmological constant. The  $D3 - \overline{D3}$  inflation of [1] can be understood as an effective description of a  $D3$  brane probe dynamics in  $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6$  warped string theory flux background. To identify the inflaton field let us focus on the throat geometry of  $\mathcal{M}_6$ . Locally, the ten dimensional metric is

$$\begin{aligned} ds_{\mathcal{M}_{10}}^2 &= \Omega_1^2 \left( r, \frac{y_i}{r} \right) ds_{\mathcal{M}_4}^2(x) + \Omega_2^2 \left( r, \frac{y_i}{r} \right) ds_{\mathcal{M}_6}^2(y) \\ &= \Omega_1^2 \left( r, \frac{y_i}{r} \right) (dS_4)^2 + \Omega_2^2 \left( r, \frac{y_i}{r} \right) dr^2 + ds_5^2 \left( r, \frac{y_i}{r} \right) + \sum_{i=1}^6 g_{ri} \left( r, \frac{y_i}{r} \right) dr dy^i, \end{aligned} \quad (3.2)$$

where  $r$  is the “radial” coordinate along the throat, roughly  $r^2 \sim \sum y_i^2$ ,  $ds_5^2$  is the “angular” part of the metric of  $\mathcal{M}_6$ , and the four dimensional de-Sitter space  $\mathcal{M}_4$  has Ricci curvature  $12H^2$ . In the “throat approximation”

$$\Omega_i \left( r, \frac{y_i}{r} \right) \approx \hat{\Omega}_i(r), \quad g_{ri} \approx 0. \quad (3.3)$$

The metric ansatz approximation (3.3) implies that the four-form potential<sup>4</sup>  $C^{(4)}$  also depends on  $r$  only

$$C^{(4)} \approx \omega(r) vol_{\mathcal{M}_4}, \quad (3.4)$$

where  $vol_{\mathcal{M}_4}$  is a volume form on  $\mathcal{M}_4$ . A  $D3$  brane at  $r = r_1$  in the throat geometry (3.3), (3.4) has an effective 10d description

$$S_{D3} = -T_3 \int_{\mathcal{M}_4} d^4\xi \sqrt{-\hat{g}(r_1)} + T_3 \int_{\mathcal{M}_4} C^{(4)}(r_1), \quad (3.5)$$

where  $\hat{g}$  is the induced metric on the world-volume of the probe. For slowly varying  $r_1 = r_1(x)$ , we find

$$\sqrt{-\hat{g}} = \sqrt{-g} \hat{\Omega}_1^4(r_1) \left( 1 + \frac{1}{2} \hat{\Omega}_1^{-2}(r_1) \hat{\Omega}_2^2(r_1) g^{\mu\nu} \partial_\mu r_1 \partial_\nu r_1 \right), \quad (3.6)$$

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<sup>4</sup>We assume that there is a self-dual five form flux  $F_5 = (1 + \star)\mathcal{F}_5$ ,  $\mathcal{F}_5 = dC^{(4)}$ , supporting the background geometry (3.2).

where  $g_{\mu\nu} = g_{\mu\nu}(x)$  is the bulk metric of (3.1). Given (3.6), the probe action evaluates to

$$S_{D3} = \int_{\mathcal{M}_4} d^4x \sqrt{-g} \left( -\frac{1}{2} T_3 \hat{\Omega}_1^2(r_1) \hat{\Omega}_2^2(r_1) (\partial r_1)^2 - \mathcal{V}(r_1) \right), \quad (3.7)$$

where  $\mathcal{V}(r_1)$  is the  $D3$  probe potential

$$\mathcal{V}(r_1) = T_3 \left( \hat{\Omega}_1^4(r_1) - \omega(r_1) \right). \quad (3.8)$$

In a consistent Kaluza-Klein reductions, the four dimensional effective description of a probe is given by the *same* effective action<sup>5</sup> as (3.5), or in the slow roll approximation (3.7). Thus identifying the effective inflaton field  $\phi$  with the radial coordinate  $r_1$  of the probe brane in the throat geometry, or more precisely

$$d\phi \equiv \sqrt{T_3} \hat{\Omega}_1(r_1) \hat{\Omega}_2(r_1) dr_1, \quad \mathcal{V}_{inf}(\phi) = \mathcal{V}(r_1(\phi)), \quad (3.9)$$

we obtain the following effective inflaton action

$$S_{inf} = \int_{\mathcal{M}_4} d^4x \sqrt{-g} \left( -\frac{1}{2} (\partial\phi)^2 - \mathcal{V}_{inf}(\phi) \right). \quad (3.10)$$

Altogether, the complete action  $S_{cosm}$  describing the cosmology of [1] is obtained from (3.1), (3.10)

$$S_{cosm} = S_{bulk} + S_{inf}. \quad (3.11)$$

Notice that, because of gauge invariance,  $\omega(r_1)$  (and thus  $\mathcal{V}_{inf}(\phi)$ ) are defined up to an arbitrary additive constant. In what follows we fix this constant in such a way that

$$\mathcal{V}_{inf}(\phi) \Big|_{\phi=0} = 0. \quad (3.12)$$

Eq.(3.12) implies that the scale of inflation is given by the bulk Hubble parameter  $H$ . In particular, the usual slow-roll parameters  $\epsilon, \eta$  are given by

$$\begin{aligned} \epsilon &= \frac{k_4^2}{18} \left( \frac{V'_{inf}}{H^2} \right)^2, \\ \eta &= \frac{1}{3} \frac{V''_{inf}}{H^2}, \end{aligned} \quad (3.13)$$

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<sup>5</sup>This has been implicitly assumed for the  $\overline{D3}$  four-dimensional effective action in the construction of KKLT de-Sitter vacua. Details that such a prescription is indeed correct will appear in [19].



where the derivatives are with respect to  $\phi$ . An important observation is that, even though  $\epsilon$  in (3.13) depends on the four dimensional gravitational coupling<sup>6</sup>  $k_4$ , the slow-roll parameter  $\eta$  is independent of it. Thus, one can use the exact local (noncompact) description of the throat geometries to deduce model-independent predictions for  $\eta$ . For example, up to the comment in footnote 2, the exact local throat geometry for the class of inflationary models considered in [1] was presented in [11, 12].

The rest of this section is organized as follows. We first demonstrate that the leading contribution  $\eta = \frac{2}{3}$  follows from the dynamics of a  $D3$  probe brane in the exact  $AdS_5 \times T^{1,1}$  throat, where  $AdS_5$  space has four-dimensional de-Sitter slicing of curvature  $12H^2$ . We then proceed to study probe dynamics in the de-Sitter deformed KT solutions presented in [11, 12]. As advocated in [12], de-Sitter deformations of the supergravity RG flows can be interpreted as holographic dual to four dimensional gauge theories in  $dS_4$  background space-time. The fact that  $\eta = \frac{2}{3}$  for the de-Sitter deformed KW solution [14] simply follows from the fact that the inflaton field  $\phi$  is holographically dual to the conformally coupled scalar field  $\Phi$  of the  $SU(N) \times SU(N)$  KW gauge theory, which necessarily has a coupling<sup>7</sup>

$$V_\Phi = \frac{1}{12} R_4 \Phi^2 = H^2 \Phi^2, \quad (3.14)$$

where  $R_4 = 12H^2$  is the Ricci scalar of the background de-Sitter space-time of the deformed KW gauge theory. The KT/KS gauge theory [10, 7] can be understood as a deformation of the KW gauge theory  $SU(N) \times SU(N) \rightarrow SU(N+M) \times SU(N)$ , which induces the effective running of  $N$  with scale [10, 20]

$$N = N(\mu) = N_0 + \frac{3}{2\pi} g_s M^2 \ln \frac{\mu}{\mu_0} \equiv N_0 \left( 1 + P^2 \ln \frac{\mu}{\mu_0} \right), \quad (3.15)$$

where  $\mu_0$  is the strong coupling scale of the cascading gauge theory,  $g_s$  is the string coupling<sup>8</sup>, and  $P^2 \equiv 3M^2/(2\pi N_0)$ . We will study two different regimes in the de-Sitter deformed KT gauge theory<sup>9</sup>

$$\begin{aligned} (a) : \quad H \gg \mu_0 & \iff 0 < P^2 \ln \frac{\mu}{\mu_0} \ll 1, \\ (b) : \quad H \ll \mu_0 & \iff P^2 \ln \frac{\mu}{\mu_0} \gg 1. \end{aligned} \quad (3.16)$$

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<sup>6</sup>This implies, in particular, that  $\epsilon$  depends on the details of the compactification (*i.e.*, volume) to the extent that scales  $k_4^{-1}$  and the Hubble scale  $H$  can be correlated.

<sup>7</sup>Analogous observation was also made in [1].

<sup>8</sup>We set  $g_s^{-1} \equiv 4\pi(g_1^{-2}(\mu) + g_2^{-2}(\mu)) \rightarrow 1$  as  $\mu \gg \mu_0$ . In the latter equation  $g_i$  are gauge couplings of the KT gauge groups.

<sup>9</sup>The general case can be studied as well, albeit numerically.

The background geometry of the supergravity dual in the first case, (a), is the leading  $P^2$  deformation of the KW model discussed in [11], while the latter case is the leading  $H^2$  deformation of the KT background geometry which we discuss below. In each case we find correspondingly

$$\begin{aligned} (a) : \quad \eta &= \frac{2}{3} \left( 1 - \frac{P^2}{12} \ln \frac{\phi^2}{L^4 H^2 T_3} \right), \quad 0 < \frac{P^2}{12} \ln \frac{\phi^2}{L^4 H^2 T_3} \ll 1, \quad \frac{L^4 H^2 T_3}{\phi^2} \ll 1, \\ (b) : \quad \eta &= \frac{2}{3} \left( 1 + \mathcal{O} \left( \frac{L^8 H^4 T_3^2}{\phi^4} \right) \right), \quad \frac{P^2}{12} \ln \frac{\phi^2}{L^4 H^2 T_3} \gg 1, \quad \frac{L^4 H^2 T_3}{\phi^2} \ll 1, \end{aligned} \quad (3.17)$$

where  $L = 4\pi N_0(\alpha')^{\frac{27}{16}}$ . The computation of  $\eta$  in case (b) suggests that it is unlikely that one can achieve (in a computationally controllable way)  $\eta \ll 1$  for inflationary models in throat geometries of the KKLT de-Sitter vacua. The point is simply that to have computational control one has to study  $D3$  probes far from the IR end of the Klebanov-Strassler throat, or to be in the regime (b). There is a physical reason for this as well. Following KKLT construction, there is a  $\overline{D3}$  brane at the end of the KS throat, and thus bringing a  $D3$  probe close to it will cause strong  $D3 - \overline{D3}$  attraction and the effective four-dimensional inflaton will start rolling too fast.

### 3.1 Inflation in de-Sitter deformed KW geometry

Following [12], the throat geometry in this case is given by

$$ds_{10}^2 = \frac{r^2}{L^2} (-dt^2 + e^{2Ht} d\vec{x}^2) + \frac{L^2 dr^2}{L^4 H^2 + r^2} + L^2 ds_{T^{1,1}}^2, \quad (3.18)$$

where  $(ds_{T^{1,1}})^2$  is the standard metric on  $T^{1,1} = (SU(2) \times SU(2))/U(1)$  and

$$L^4 = 4\pi g_s N(\alpha')^{\frac{27}{16}}, \quad (3.19)$$

with  $N$  being the number of  $D3$  branes. The metric (3.18) is supported by the following 5-form flux

$$F_5 = \mathcal{F}_5 + \star \mathcal{F}_5, \quad \mathcal{F}_5 = -4L^4 d\text{vol}_{T^{1,1}}. \quad (3.20)$$

From (3.20) we conclude that

$$dC^{(4)} = \frac{4r^4}{L^4 \sqrt{L^4 H^2 + r^2}} dr \wedge d\text{vol}_{dS_4}. \quad (3.21)$$

Using (3.8), (3.18), (3.21) we find

$$\begin{aligned}
T_3^{-1}\mathcal{V}(r_1) &= \frac{r_1^4}{L^4} \left( 1 - \sqrt{1 + \frac{L^4 H^2}{r_1^2}} \right) + \frac{3}{2} H^2 r_1^2 \sqrt{1 + \frac{L^4 H^2}{r_1^2}} \\
&\quad - \frac{3}{2} L^4 H^4 \ln \frac{r_1 + \sqrt{L^4 H^2 + r_1^2}}{L^2 H} \\
&= H^2 r_1^2 \left( 1 + \mathcal{O} \left( \frac{L^4 H^2}{r_1^2} \right) \right) .
\end{aligned} \tag{3.22}$$

Defining an effective inflaton field as in (3.9)

$$d\phi = \sqrt{T_3} \frac{r_1}{\sqrt{L^4 H^2 + r_1^2}} dr_1 , \tag{3.23}$$

we find

$$\begin{aligned}
\mathcal{V}_{inf}(\phi) &= H^2 \phi^2 \left( 1 + \mathcal{O} \left( \frac{L^4 H^2 T_3}{\phi^2} \right) \right) \\
&\approx H^2 \phi^2 ,
\end{aligned} \tag{3.24}$$

where in the second line we imposed constraint for inflation to occur far from the IR end of the throat

$$\frac{L^4 H^2 T_3}{\phi^2} \ll 1 . \tag{3.25}$$

With (3.13), (3.24) we reproduce

$$\eta = \frac{2}{3} . \tag{3.26}$$

### 3.2 Inflation in de-Sitter deformed KT geometry

Without loss of generality we set  $L = 1$ . From [11,12] we extract the exact background describing the throat geometry

$$ds_{10}^2 = e^{2z} (dM_4^H)^2 + e^{-2z} [e^{10y} du^2 + e^{2y} (dM_5)^2] . \tag{3.27}$$

Here  $M_5$  is a deformation of the  $T^{1,1}$  metric

$$\begin{aligned}
(dM_5)^2 &= e^{-8w} e_\psi^2 + e^{2w} (e_{\theta_1}^2 + e_{\phi_1}^2 + e_{\theta_2}^2 + e_{\phi_2}^2) , \\
e_\psi &= \frac{1}{3} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2) , \quad e_{\theta_i} = \frac{1}{\sqrt{6}} d\theta_i , \quad e_{\phi_i} = \frac{1}{\sqrt{6}} \sin \theta_i d\phi_i ,
\end{aligned} \tag{3.28}$$

and

$$(dM_4^H)^2 = -dt^2 + e^{2Ht} d\vec{x}^2 . \tag{3.29}$$

As for the matter fields, we have a dilaton  $\Phi$  that depends on the radial coordinate  $u$  only, and the 3- and 5-form fluxes

$$\begin{aligned} F_3 &= P e_\psi \wedge (e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2}) , & B_2 &= f(u)(e_{\theta_1} \wedge e_{\phi_1} - e_{\theta_2} \wedge e_{\phi_2}) , \\ F_5 &= \mathcal{F} + *\mathcal{F} , & \mathcal{F} &= K(u) e_\psi \wedge e_{\theta_1} \wedge e_{\phi_1} \wedge e_{\theta_2} \wedge e_{\phi_2} , & K(u) &= 4 + 2P f(u) . \end{aligned} \quad (3.30)$$

The corresponding system of type IIB supergravity equations of motion describing the radial evolution of the five unknown functions of  $u$ :  $y, z, w, K, \Phi$  is given by [12]

$$\begin{aligned} 10y'' - 8e^{8y}(6e^{-2w} - e^{-12w}) - 30H^2 e^{10y-4z} + \Phi'' &= 0 , \\ 10w'' - 12e^{8y}(e^{-2w} - e^{-12w}) - \Phi'' &= 0 , \\ \Phi'' + e^{-\Phi+4z-4y-4w} \left( \frac{K'^2}{4P^2} - e^{2\Phi+8y+8w} P^2 \right) &= 0 , \\ 4z'' - K^2 e^{8z} - e^{-\Phi+4z-4y-4w} \left( \frac{K'^2}{4P^2} + e^{2\Phi+8y+8w} P^2 \right) - 12H^2 e^{10y-4z} &= 0 , \\ (e^{-\Phi+4z-4y-4w} K')' - 2P^2 K e^{8z} &= 0 , \end{aligned} \quad (3.31)$$

with the first order constraint

$$\begin{aligned} 5y'^2 - 2z'^2 - 5w'^2 - \frac{1}{8}\Phi'^2 - \frac{1}{4}e^{-\Phi+4z-4y-4w} \frac{K'^2}{4P^2} \\ - 3H^2 e^{10y-4z} - e^{8y}(6e^{-2w} - e^{-12w}) + \frac{1}{4}e^{\Phi+4z+4y+4w} P^2 + \frac{1}{8}e^{8z} K^2 &= 0 . \end{aligned} \quad (3.32)$$

In eqs. (3.31), (3.32) prime denotes derivative with respect to  $u$ .

Rather than computing the  $D3$  probe brane potential (3.8) in the above geometry, it is convenient to compute its derivative with respect to the radial coordinate  $u$ . We find

$$\frac{d\mathcal{V}(u)}{du} = T_3 e^{4z} \left( 4z' + K e^{4z} \right) . \quad (3.33)$$

Also, using (3.9), the effective inflaton field  $\phi$  is defined as

$$d\phi = -\sqrt{T_3} e^{5y} du . \quad (3.34)$$

### 3.2.1 Case (a)

In order to make use of the results of [11] we set  $H = 1$ . The  $H$  dependence in the final expression for  $\eta$  can be restored from dimensional analysis. It will also be convenient to introduce a new radial coordinate  $\rho$  such that

$$d\rho = -e^{4y} du . \quad (3.35)$$

The  $\mathcal{O}(P^2)$  solution to (3.31) can be parameterized as

$$\begin{aligned} K &= 4 + 2P^2 k(\rho) , & \Phi &= P^2 \hat{\Phi}(\rho) , & w &= P^2 \omega(\rho) , \\ y &= y_0(\rho) + P^2 \xi(\rho) , & z &= y_0(\rho) + P^2 \eta(\rho) , & y_0(\rho) &\equiv \ln \sinh \rho . \end{aligned} \quad (3.36)$$

Thus, using (3.35) we find from (3.33)

$$\begin{aligned} T_3^{-1} \frac{d\mathcal{V}(\rho)}{d\rho} &= 4e^{4y_0} \left\{ y_0' (1 + 4P^2 \eta) + P^2 \eta' - 1 - P^2 \left( \frac{k}{2} + 8\eta - 4\xi \right) + \mathcal{O}(P^4) \right\} \\ &= 4e^{4y_0} \left\{ P^2 \left( \eta' + 4\xi - 4\eta - \frac{k}{2} \right) + \frac{1}{2} e^{-2y_0} (1 + 4P^2 \eta) + \mathcal{O}(e^{-4y_0}) \right\} + \mathcal{O}(P^4) , \end{aligned} \quad (3.37)$$

where the derivatives are taken with respect to  $\rho$  now. In the second line in (3.37) we used

$$y_0' = \sqrt{1 + e^{-2y_0}} = 1 + \frac{1}{2} e^{-2y_0} + \mathcal{O}(e^{-4y_0}) . \quad (3.38)$$

Introducing  $\nu \equiv 5\xi - 2\eta$  we find

$$\begin{aligned} \eta' + 4\xi - 4\eta - \frac{k}{2} &= -\frac{1}{2} (\nu' - 4\nu + k) + \frac{5}{2} \xi' - 6\xi \\ &= \frac{1}{3} e^{-2y_0} \nu + \dots \\ &= \frac{1}{12} e^{-2y_0} \rho + \dots , \end{aligned} \quad (3.39)$$

where we used eqs. (4.30), (4.31), (4.36) of ref. [11], and the asymptotics (eq. (4.18) of ref. [11])

$$\begin{aligned} \nu &\rightarrow \frac{1}{4} \rho , & \rho &\rightarrow \infty , \\ \eta &\rightarrow -\frac{1}{8} \rho , & \rho &\rightarrow \infty , \\ \xi &\rightarrow -\frac{1}{6} e^{-2y_0} \nu , & \rho &\rightarrow \infty . \end{aligned} \quad (3.40)$$

The ellipses in (3.39) denote subdominant terms as  $\rho \rightarrow \infty$ . With (3.39), (3.40) we conclude from (3.37)

$$T_3^{-1} \frac{d\mathcal{V}(\rho)}{d\rho} = 2e^{2y_0} \left( 1 - P^2 \left( \frac{\rho}{6} + \mathcal{O}(1) \right) + \mathcal{O}(e^{-2y_0}) \right) + \mathcal{O}(P^4) . \quad (3.41)$$

Using (3.35), the inflaton is defined according to (3.34)

$$\begin{aligned} T_3^{-1/2} d\phi &= e^y d\rho \\ &= e^{y_0} (1 + P^2 \xi + \mathcal{O}(P^4)) d\rho \\ &= e^{y_0} \left( 1 - \frac{P^2}{24} e^{-2y_0} \rho + \mathcal{O}(P^4) \right) d\rho , \end{aligned} \quad (3.42)$$

where in the last line we used asymptotics (3.40). Thus

$$T_3^{-1/2}\phi = \frac{1}{2}e^\rho (1 + \mathcal{O}(e^{-2\rho})) , \quad (3.43)$$

which upon integration of (3.41) yields to leading order in  $P^2$  and  $T_3/\phi^2$

$$\mathcal{V}_{inf}(\phi) = \phi^2 \left( 1 - \frac{P^2}{12} \ln \frac{\phi^2}{T_3} \right) , \quad (3.44)$$

which leads to the slow-roll parameter reported in (3.17), case (a).

### 3.2.2 Case (b)

In this case we need to find the  $\mathcal{O}(H^2)$  solution to (3.31) around KT solution [10]. Here, it is convenient to use  $u$  as a radial coordinate. Analogously to the previous case we search for a solution in the following parametrization

$$\begin{aligned} K &= K_{KT} + H^2 K_1 , & \Phi &= \Phi_{KT} + H^2 \Phi_1 , & w &= w_{KT} + H^2 w_1 , \\ y &= y_{KT} + H^2 y_1 , & z &= z_{KT} + H^2 z_1 , \end{aligned} \quad (3.45)$$

where the subscript  $_{KT}$  denotes the KT solution (see eqs. (3.9), (3.10) of ref. [11])

$$\begin{aligned} w_{KT} &= \Phi_{KT} = 0 , & e^{-4y_{KT}} &= 4u , & K_{KT} &= 4 - \frac{P^2}{2} \ln u , \\ e^{-4z_{KT}} &= \left( 4 + \frac{P^2}{2} \right) u - \frac{P^2}{2} u \ln u . \end{aligned} \quad (3.46)$$

Given the parametrization (3.45) we deduce from (3.33)

$$T_3^{-1} \frac{d\mathcal{V}(u)}{du} = 4H^2 e^{4z_{KT}} \left( z_1' + e^{4z_{KT}} \left( z_1 K_{KT} + \frac{1}{4} K_1 \right) \right) + \mathcal{O}(H^4) . \quad (3.47)$$

As in the example studied in [11], it is possible to expand the system of equations (3.31) around the KT solution (3.46) to  $\mathcal{O}(H^2)$  order and obtain a coupled system for the deformations  $\{K_1, \Phi_1, w_1, y_1, z_1\}$ . The resulting equations are rather complicated, and we will not present them here. Rather miraculously, it turns out that the differential equation for

$$h(u) \equiv z_1' + e^{4z_{KT}} \left( z_1 K_{KT} + \frac{1}{4} K_1 \right) \quad (3.48)$$

decouples from the other equations

$$0 = h' - e^{4z_{KT}} K_{KT} h - 3e^{10y_{KT}-4z_{KT}} . \quad (3.49)$$

Moreover, given (3.46), it can be solved exactly

$$h(u) = u \left( P^2(\ln u - 1) - 8 \right) \left( c + \frac{1}{32u^{3/2}} \right), \quad (3.50)$$

where  $c$  is an arbitrary integration constant. From (3.47) we find then

$$T_3^{-1} \frac{d\mathcal{V}(u)}{du} = -H^2 \left( \frac{1}{4u^{3/2}} + 8c \right) + \mathcal{O}(H^4). \quad (3.51)$$

From (3.34) we find

$$\phi = \sqrt{\frac{T_3}{2}} u^{-1/4} + \mathcal{O}(H^2). \quad (3.52)$$

Finally, to leading order in  $H^2$  and  $T_3/\phi^2$  we find from (3.51), (3.52)

$$\mathcal{V}_{inf}(\phi) = H^2 \phi^2, \quad (3.53)$$

which leads to the slow-roll parameter reported in (3.17), case (b).

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